OFDM

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Single carrier vs. multiple carrier

- Single carrier
  - Large # of channel taps
  - Frequency selective
  - Challenge for both channel estimation and equalization
- Multicarrier
  - Convert frequency selective to flat equivalent

Block-wise transmission

$$\bar{\mathbf{x}}(i) = \mathbf{H}_0\bar{\mathbf{u}}(i) + \mathbf{H}_1\bar{\mathbf{u}}(i-1) + \bar{\boldsymbol{\eta}}(i),$$

where $\bar{\eta}(i)$ is the corresponding noise vector, and for $l = 0, 1$, the $P \times P$ matrices $\mathbf{H}_l$ are defined to have the (i, j)th entry as $h(lP + i - j)$; i.e.,

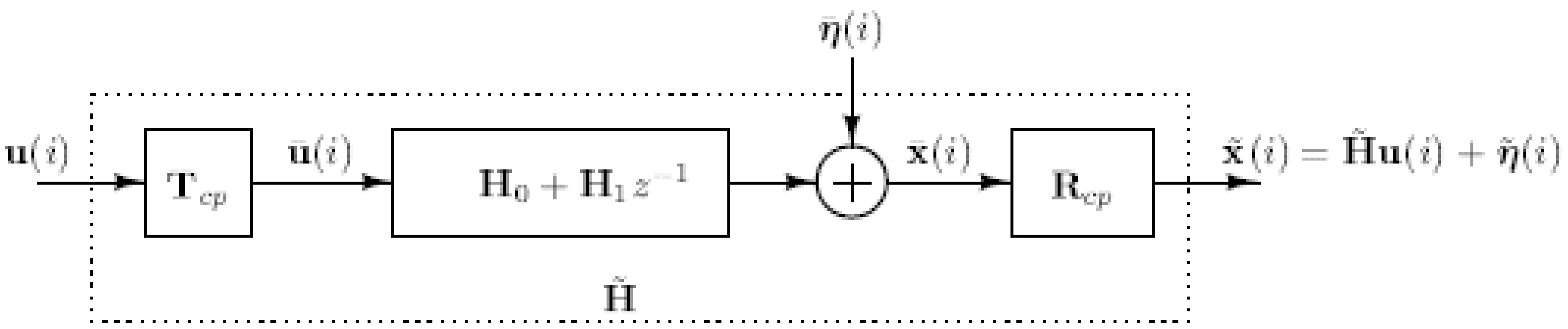
$$\mathbf{H}_0 = \begin{pmatrix} h(0) & 0 & 0 & \cdots & 0 \\ \vdots & h(0) & 0 & \cdots & 0 \\ h(L) & \cdots & \ddots & \cdots & \vdots \\ \vdots & \ddots & \cdots & \ddots & 0 \\ 0 & \cdots & h(L) & \cdots & h(0) \end{pmatrix},$$

$$\mathbf{H}_1 = \begin{pmatrix} 0 & \cdots & h(L) & \cdots & h(1) \\ \vdots & \ddots & 0 & \ddots & \vdots \\ 0 & \cdots & \ddots & \cdots & h(L) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 \end{pmatrix}.$$

Inserting guard interval

$$\bar{\mathbf{u}}(i) = \mathbf{T}\mathbf{u}(i)$$

$\mathbf{T}$ is $P \times N$, with $P = N + L$. $\mathbf{T} = \mathbf{T}_{cp} := [\mathbf{I}_{cp}^T \; \mathbf{I}_N^T]^T$



$$\bar{\mathbf{x}}(i) = \mathbf{H}_0\mathbf{T}\mathbf{u}(i) + \mathbf{H}_1\mathbf{T}\mathbf{u}(i-1) + \bar{\eta}(i).$$

Circulant channel equivalent

$$\tilde{\mathbf{H}} := \tilde{\mathbf{H}}_0 \mathbf{T}_{cp} = \mathbf{R}_{cp} \mathbf{H}_0 \mathbf{T}_{cp}$$

with its (k, l)th entry given by $h\big((k-l) \mod N\big)$

$$\tilde{\mathbf{x}}(i) = \tilde{\mathbf{H}} \mathbf{u}(i) + \tilde{\boldsymbol{\eta}}(i),$$

DFT matrix

$$X = Wx \qquad W = \left(\frac{\omega^{jk}}{\sqrt{N}}\right)_{j,k=0,\ldots,N-1}$$

$$W = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega & \omega^2 & \omega^3 & \cdots & \omega^{N-1} \\ 1 & \omega^2 & \omega^4 & \omega^6 & \cdots & \omega^{2(N-1)} \\ 1 & \omega^3 & \omega^6 & \omega^9 & \cdots & \omega^{3(N-1)} \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 1 & \omega^{N-1} & \omega^{2(N-1)} & \omega^{3(N-1)} & \cdots & \omega^{(N-1)(N-1)} \end{bmatrix},$$

$$\omega = e^{-\frac{2\pi i}{N}}$$

Examples

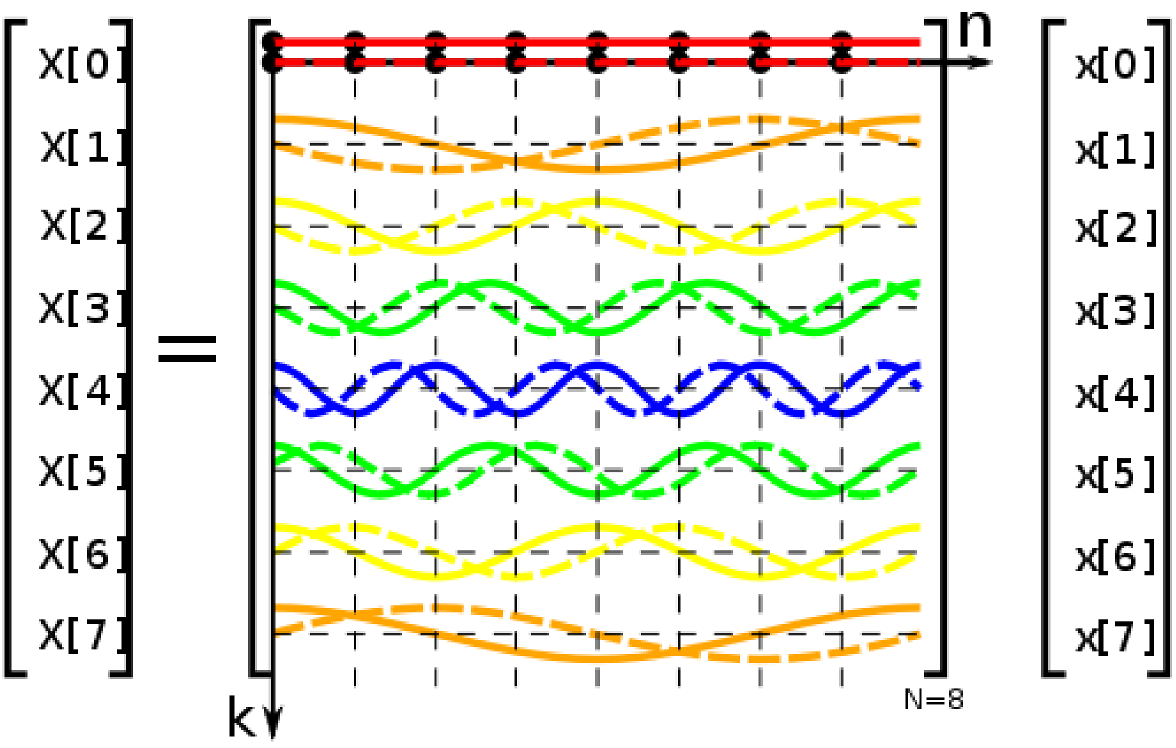
$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -i & -1 & i\\ 1 & -1 & 1 & -1\\ 1 & i & -1 & -i\end{bmatrix}.$$

$$W = \frac{1}{\sqrt{8}}\begin{bmatrix}\omega^0 & \omega^0 & \omega^0 & \dots & \omega^0\\ \omega^0 & \omega^1 & \omega^2 & \dots & \omega^7\\ \omega^0 & \omega^2 & \omega^4 & \dots & \omega^{14}\\ \omega^0 & \omega^3 & \omega^6 & \dots & \omega^{21}\\ \omega^0 & \omega^4 & \omega^8 & \dots & \omega^{28}\\ \omega^0 & \omega^5 & \omega^{10} & \dots & \omega^{35}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ \omega^0 & \omega^7 & \omega^{14} & \dots & \omega^{49}\end{bmatrix}$$

$$\omega = e^{-\frac{2\pi i}{8}} = \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$$

Illustration



Diagonalization of a circulant matrix

Fact 1: (Diagonalization of a circulant matrix) An $N \times N$ circulant matrix $\tilde{\mathbf{H}}$ can be diagonalized by pre- and post-multiplication with N-point FFT and IFFT matrices; i.e., $\mathbf{F}\tilde{\mathbf{H}}\mathbf{F}^{-1} = \mathbf{D}_H := \mathrm{diag}\left[H(e^{j0}), H(e^{j2\pi/N}), \ldots, H(e^{j2\pi(N-1)/N})\right]$, where $[\mathbf{F}]_{k,n} = N^{-\frac{1}{2}} \exp(-j2\pi kn/N)$ and $H(e^{j2\pi f}) := \sum_{n=0}^{L} h(n) \exp(-j2\pi fn)$ is the frequency response of the LTI channel.

- Equivalent diagonal channel model

Subcarriers